VOLUME 4
TEST MANAGEMENT PHASE

CHAPTER 8
DESIGN OF EXPERIMENTS
APPENDICES

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USAF TEST PILOT SCHOOL
EDWARDS AIR FORCE BASE, CALIFORNIA

APPENDIX A

COURSE OBJECTIVES

Based on USAF TPS/ED Course Training Standard, all Design of Experiments require C-Application Level Subject Knowledge (i.e. Students can use learned material in new and concrete situations).

Lesson 1: Introduction

1.1 State the major factors of test development.

1.2 Understand why clearly stating the test purpose is critical to test success.

1.3 Understand the purpose of success criteria.

1.4 Know the differences between the null hypothesis and the alternate hypothesis.

1.5 Understand the concept of acceptable risk.

1.6 Distinguish the difference between a type α error and a type β error.

1.7 Understand why sample size is critical to the design of experiments

1.8 Distinguish the difference between systemic errors and random errors.

Lesson 2: Basic Probability and Statistics

2.1 Understand concepts of classical probability and experimental probability.

2.2 Apply the concepts of the probabilities axioms to the following events: complementary, mutually exclusive, and independent.

2.3 Distinguish the difference between populations and samples.

2.4 Know the difference between homogeneous, independent, and random samples.

2.5 Understand and calculate measures of central tendency.

2.6 Understand and calculate measures of dispersion.

Lesson 3: Probability Distributions

3.1 Understand the difference between discrete and continuous probability distributions and their respective interpretations and applications.

3.2 Understand and apply the concepts associated with a discrete or continuous cumulative distribution functions.

3.3 Understand and apply the concepts associated with the binomial distribution.

3.4 Understand and apply the concepts associated with the normal distribution and standard normal distribution.

3.5 Understand and apply the concepts associated with the student's t distribution.

3.6 Understand and apply the concepts associate with the chi-square distribution.

Lesson 4: Confidence Limits

4.1 Understand and apply the concepts associated with the Central Limit Theorem.

4.2 Understand the application of the Central Limit Theorem to sampling theory.

4.3 Define the uncertainty level, α, associated with sampling theory.

4.4 Calculate confidence intervals for means for varying levels of uncertainty (α) for sample sizes $n \geq 30$.

4.5 Calculate confidence intervals for means for varying levels of uncertainty (α) for sample sizes $n < 30$.

4.6 Calculate confidence intervals for variance for varying levels of uncertainty (α) and degrees of freedom (υ).

Lesson 5: Hypothesis Testing

5.1 Understand the basic concepts underlying hypothesis testing.

5.2 Comprehend the purpose and development of the null hypothesis (H_0) and the alternative hypothesis (H_1).

5.3 Understand the concept of a Type I error and the level of significance, α.

5.4 Understand the concept of a Type II error and its probability, β.

5.5 Setup the null hypothesis accept and reject regions for one tailed and two tailed tests.

5.6 Apply one tailed and two tailed tests on means using hypothesis testing for $n \geq 30$ or σ known, using the z - test statistic.

5.7 Apply one tailed and two tailed tests on means using hypothesis testing for $n < 30$ and σ unknown, using the t - test statistic.

5.8 Apply one tailed test on variances using the chi-squared statistic.

Lesson 6: Nonparametric Testing

6.1 Know why nonparametric tests are used in place of parametric tests.

6.2 Know the nonparametric sign test assumes that two populations are equivalent.

6.3 Know the binomial distribution is the basis for the nonparametric sign test.

6.4 Apply the sign test to a comparison evaluation.

Lesson 7: Sample Size Determination

7.1 Understand the tradeoffs involved in sample size determination.

7.2 Apply the concept of confidence interval requirement to the accuracy driven sample size determination.

7.3 Apply the concept of hypothesis testing to the general approach of sample size determination.

Lesson 8: Error Analysis

8.1 Know the purpose of significant figures.

8.2 Given a number, determine the number of significant figures in the number.

8.3 Know the generally accepted rule for significant figures based on experimental precision.

8.4 Apply the rules for mathematical operations on significant figures.

8.5 Given an equation, apply the techniques of error propagation to determine the contribution of independent variable error to overall functional error.

8.6 Given an equation, apply the techniques of error propagation to determine the contribution of the independent variables variances to the overall functional variance.

APPENDIX B

PROBLEM SETS

Lesson 1

1. Determining "how much risk of being wrong is acceptable" is a major factor in test development (T/F)

2. Systemic errors are nonrepeatable errors caused by a flaw in a measuring system (T/F)

3. Clearly stating the purpose of the test is necessary for:
   a. Creating generalized success criteria
   b. Providing focus on a specified goal
   c. Determining the maximum number of samples needed to complete the test
   d. a and c
   e. All of the above

Lesson 2

1. A data sample is any subset of a given population (T/F)

2. A random sample is one where the selection of one data point does not affect the likelihood of subsequent data points (T/F)

3. Given a fair deck of 52 cards

    Event A: Pick 1 King

    Event B: Compliment of Event A

    The probability of Event B is:

    a. 4/52
    b. 1/52
    c. 51/52
    d. 48/52
    e. None of the above

4. The mean of a sample is:

    a. The most common measure of central tendency
    b. Arithmetic average
    c. a and b
    d. All of the above
    e. None of the above

5. Your data group has been asked to verify the takeoff performance in the T-38. Your group decides to do 10 takeoffs all on the same day in the same aircraft without refueling between takeoffs. All 5 pilots want to fly, so it is decided to let each pilot do 2 takeoffs. Are your data:

    a. homogeneous
    b. independent
    c. random

6. Two cards are drawn from a single deck. Find the probability that they are both aces if the first card is:

    a. replaced
    b. not replaced

7. Given the following random independent 360° aileron roll data:

Test Point	Time
1	3.5
2	4.0
3	3.8
4	4.2
5	3.7

Find

a. Sample mean

b. Sample median

c. Sample standard deviation

8. Top Pilot Mickey Mach decides to separate from the Air Force and try his golden hand at professional baseball. Halfway through his rookie year, Mickey has a 0.331 batting average and has at least 1 hit in each of the past 10 games. If the league record is hitting 41 games in a row, what is the probability of setting a new record? Assume Mickey gets up to bat 5 times in each game and each time up to bat is independent of the others.

Lesson 3

1. The Binomial distribution is an example of a continuous probability distribution (T/F)

2. The Chi-squared distribution is used when comparing sample means and population means (T/F)

3. The probability of getting a total of 7 only once in three tosses of a pair of fair dice is:

   a. $6\ (1/6)^2\ (5/6)$

   b. $3\ (1/6)\ (5/6)^2$

   c. $3\ (1/6)^2\ (5/6)$

   d. $6\ (1/6)\ (5/6)^2$

   e. None of the above

4. The value of Z such that the area between -1.5 and Z is .0013 is:

   a. $Z = -1.49$

   b. $Z = -1.51$

   c. $Z = -3.00$

   d. a or b

   e. None of the above

5. Over a long period of time the airlines noticed that only 90% of the people making reservations actually showed up. The standard deviation is 4% of those making reservations. Assume that these data represent true population values and that the data is normally distributed.

a. What is the probability of too many people showing up for a single flight if 105% of the seats are reserved?

b. What is the probability of getting an average of only 85% or less of the people to show up over a sample of 5 flights (assume $S = \sigma$)?

c. Ninety-nine percent of the time, you should expect standard deviation in b to be less than what value?

Lesson 4

1. The central limit theorem requires that the population from which a sample is taken be normally distributed (T/F)

2. Confidence intervals for variance are based on student's t distributions (T/F)

3. Given a general population with mean μ_A and standard deviation σ_A , and a sample mean $\mu_{\bar{x}}$ and a standard deviation $\sigma_{\bar{x}}$, for a sample size equal to infinity then.

   a. $\mu_{\bar{x}} = \mu_A$
   b. $\mu_x = 0$
   c. $\sigma_{\bar{x}} = \sigma_A$
   d. $\sigma_{\bar{x}} = 0$
   e. a and d

4. For sample sizes greater or equal to 30, confidence intervals are based on:

   a. Binomial distribution
   b. Standard normal distribution
   c. Student's t distribution
   d. Chi-squared distribution
   e. None of the above

5. Ten MIL power takeoff rolls were measured by your data group. The standardized data have a mean of 2700 ft and a standard deviation of 200 ft. What are the 95% confidence limits for the actual value?.

Lesson 5

1. A type II error occurs when we accept the null hypothesis and it is false (T/F)

2. For $n < 30$ and σ is unknown, a test of means should be based on a standard normal distribution statistic (T/F)

3. The failure to reject the null hypothesis is due to:
   a. The fact that it is true
   b. Insufficient evidence to reject it
   c. The fact that the alternative hypothesis is false
   d. a and c
   e. None of the above

4. Rocket motors have burn times of 3 sec (μ_0) when produced. A sample of 9 motors which were stored for 5 years had an average burn time of 3.1 sec and a standard deviation of .1 sec. At the 95% confidence level, has the burn changed.

5. The specification for engine thrust on the F-120 is 28,000 lbs. You test 11 engines and find the mean is 27,500 with a standard deviation of 350 lbs. Did the contractor meet the specification at the 90% level of significance.

Lesson 6

1. The YF-19 has vertical tape instruments. Before going into production, the SPO Director polls the test pilots and finds that 10 prefer round dials, 2 have no preference and 5 want to keep the tapes. You want to be 95% sure before approving an engineering change proposal (ECP). What should you do?

2. The YF-23 handling qualities were preferred by 4 of 11 pilots. The YF-22 handling qualities were preferred by 6 of the same 11 pilots. One had no preference.

   a. If the two handling qualities are truly equal, what is the probability of 6 or more test pilots would prefer the YF-22 handling qualities.

   b. Based on this sample, should the Advanced Tactical fighter evaluation team credit the YF-22 as the winner in handling qualities with 90% confidence.

Lesson 7

1. A 100 percent sampling of a population is the most cost effective means of testing (T/F)

2. Keeping other factors fixed in the general approach to sample size determination, then increasing α, leads to increased β (T/F)

3. For the accuracy driven sample size determination, sample size increases as:
   a. Z test statistic increases
   b. α decreases
   c. Population variance increases
   d. All of the above
   e. None of the above

4. How many samples do we need to determine the mean at the 95% confidence level if we want the error to be:

   a. Less than .1 σ
   b. Less than .2 σ
   c. Less than .3 σ

5. A new radar component is being tested to determine its effect on detection range. From previous tests, the standard deviation of such tests is about 1.5 nm. How many test points must we fly with both the old and new component if we want to detect a mean difference of 1 NM at 95% confidence, while guarding against the false positive with probability 90%.

Lesson 8

1. If there is no decimal point, the rightmost nonzero digit is the least significant digit (T/F)

2. The number of significant digits in the number 0.0020300 is:
   a. 3
   b. 4
   c. 5
   d. 7
   e. 8

3. The answer containing the correct number of significant digits for 1.22 + 3.5678 + 4.534 is:
   a. 9.321
   b. 9.32
   c. 9.322
   d. 9.3218
   e. None of the above

4. To what fractional accuracy (%) can we specify the volume of a sphere ($V = 4/3\ \pi\ r^3$) if we can measure the radius to within 1%

ANSWERS

1-1	T
1-2	F
1-3	b
2-1	T
2-2	F
2-3	d
2-4	c
2-5a	Yes, if all pilots use the same procedure, technique, etc
2-5b	Yes
2-5c	Yes, if corrected to standard weight and atmospheric conditions
2-6a	.00592
2-6b	.00452
2-7a	3.84
2-7b	3.80
2-7c	.270
2-8	$(.866)^{32} = .01 = 1\%$
3-1	F
3-2	F
3-3	b
3-4	d
3-5a	9.5%
3-5b	2.5%
3-5c	7.3%
4-1	F
4-2	F
4-3	e
4-4	b
4-5	$2557 \leq \mu \leq 2843$
5-1	T
5-2	F
5-3	b
5-4	Yes
5-5	No
6-1	Keep the tapes
6-2a	.3770
6-2b	No
7-1	F
7-2	F
7-3	d
7-4a	385
7-4b	97
7-4c	43
7-5	20 for each system
8-1	T
8-2	c
8-3	c
8-4	3%

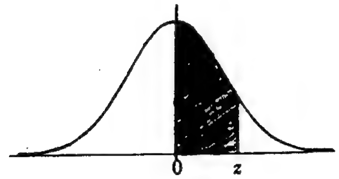
APPENDIX C

n N	0	1	2	3	4	5	6	7	8	9	10	11	12	13
2	.250	.500	.250											
3	.125	.375	.375	.125										
4	.062	.250	.375	.250	.062									
5	.031	.156	.312	.312	.156	.031								
6	.016	.094	.234	.312	.234	.094	.016							
7	.008	.055	.164	.273	.273	.164	.055	.008						
8	.004	.031	.109	.219	.273	.219	.109	.031	.004					
9	.002	.018	.070	.164	.246	.246	.164	.070	.018	.002				
10	.001	.010	.044	.117	.205	.246	.205	.117	.044	.010	.001			
11		.005	.027	.081	.161	.226	.226	.161	.081	.027	.005			
12		.003	.016	.054	.121	.193	.226	.193	.121	.054	.016	.003		
13		.002	.010	.035	.087	.157	.209	.209	.157	.087	.035	.010	.002	
14		.001	.006	.022	.061	.122	.183	.209	.183	.122	.061	.022	.006	.001

Table of Binomial Probabilities for p=q=.5

APPENDIX D

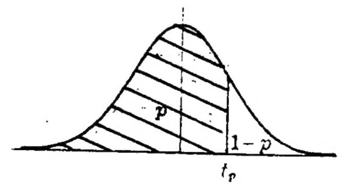
Areas under the Standard Normal Curve from 0 to z



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

APPENDIX E

Percentile Values (t_p)
for
Student's t Distribution
with ν Degrees of Freedom

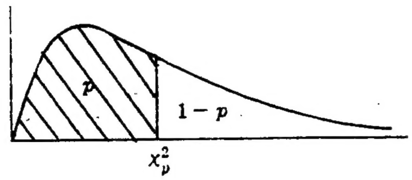


ν	$t_{.55}$	$t_{.60}$	$t_{.70}$	$t_{.75}$	$t_{.80}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$
1	.158	.325	.727	1.000	1.376	3.08	6.31	12.71	31.82	63.66
2	.142	.289	.617	.816	1.061	1.89	2.92	4.30	6.96	9.92
3	.137	.277	.584	.765	.978	1.64	2.35	3.18	4.54	5.84
4	.134	.271	.569	.741	.941	1.53	2.13	2.78	3.75	4.60
5	.132	.267	.559	.727	.920	1.48	2.02	2.57	3.36	4.03
6	.131	.265	.553	.718	.906	1.44	1.94	2.45	3.14	3.71
7	.130	.263	.549	.711	.896	1.42	1.90	2.36	3.00	3.50
8	.130	.262	.546	.706	.889	1.40	1.86	2.31	2.90	3.36
9	.129	.261	.543	.703	.883	1.38	1.83	2.26	2.82	3.25
10	.129	.260	.542	.700	.879	1.37	1.81	2.23	2.76	3.17
11	.129	.260	.540	.697	.876	1.36	1.80	2.20	2.72	3.11
12	.128	.259	.539	.695	.873	1.36	1.78	2.18	2.68	3.06
13	.128	.259	.538	.694	.870	1.35	1.77	2.16	2.65	3.01
14	.128	.258	.537	.692	.868	1.34	1.76	2.14	2.62	2.98
15	.128	.258	.536	.691	.866	1.34	1.75	2.13	2.60	2.95
16	.128	.258	.535	.690	.865	1.34	1.75	2.12	2.58	2.92
17	.128	.257	.534	.689	.863	1.33	1.74	2.11	2.57	2.90
18	.127	.257	.534	.688	.862	1.33	1.73	2.10	2.55	2.88
19	.127	.257	.533	.688	.861	1.33	1.73	2.09	2.54	2.86
20	.127	.257	.533	.687	.860	1.32	1.72	2.09	2.53	2.84
21	.127	.257	.532	.686	.859	1.32	1.72	2.08	2.52	2.83
22	.127	.256	.532	.686	.858	1.32	1.72	2.07	2.51	2.82
23	.127	.256	.532	.685	.858	1.32	1.71	2.07	2.50	2.81
24	.127	.256	.531	.685	.857	1.32	1.71	2.06	2.49	2.80
25	.127	.256	.531	.684	.856	1.32	1.71	2.06	2.48	2.79
26	.127	.256	.531	.684	.856	1.32	1.71	2.06	2.48	2.78
27	.127	.256	.531	.684	.855	1.31	1.70	2.05	2.47	2.77
28	.127	.256	.530	.683	.855	1.31	1.70	2.05	2.47	2.76
29	.127	.256	.530	.683	.854	1.31	1.70	2.04	2.46	2.76
30	.127	.256	.530	.683	.854	1.31	1.70	2.04	2.46	2.75
40	.126	.255	.529	.681	.851	1.30	1.68	2.02	2.42	2.70
60	.126	.254	.527	.679	.848	1.30	1.67	2.00	2.39	2.66
120	.126	.254	.526	.677	.845	1.29	1.66	1.98	2.36	2.62
∞	.126	.253	.524	.674	.842	1.28	1.645	1.96	2.33	2.58

Source: R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, published by Longman Group Ltd., London (previously published by Oliver and Boyd, Edinburgh), and by permission of the authors and publishers.

APPENDIX F

Percentile Values (χ^2_p) for the Chi-Square Distribution with ν Degrees of Freedom



ν	$\chi^2_{.005}$	$\chi^2_{.01}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.10}$	$\chi^2_{.25}$	$\chi^2_{.50}$	$\chi^2_{.75}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$	$\chi^2_{.999}$
1	.0000	.0002	.0010	.0039	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63	7.88	10.8
2	.0100	.0201	.0506	.103	.211	.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6	13.8
3	.0717	.115	.216	.352	.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8	16.3
4	.207	.297	.484	.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9	18.5
5	.412	.554	.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7	20.5
6	.676	.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5	22.5
7	.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3	24.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0	26.1
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6	27.9
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2	29.6
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8	31.3
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3	32.9
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8	34.5
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3	36.1
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8	37.7
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3	39.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7	40.8
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2	42.3
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6	43.8
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0	45.3
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4	46.8
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8	48.3
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2	49.7
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6	51.2
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9	52.6
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3	54.1
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6	55.5
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0	56.9
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3	58.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7	59.7
40	20.7	22.2	24.4	26.5	29.1	33.7	39.3	45.6	51.8	55.8	59.3	63.7	66.8	73.4
50	28.0	29.7	32.4	34.8	37.7	42.9	49.3	56.3	63.2	67.5	71.4	76.2	79.5	86.7
60	35.5	37.5	40.5	43.2	46.5	52.3	59.3	67.0	74.4	79.1	83.3	88.4	92.0	99.6
70	43.3	45.4	48.8	51.7	55.3	61.7	69.3	77.6	85.5	90.5	95.0	100	104	112
80	51.2	53.5	57.2	60.4	64.3	71.1	79.3	88.1	96.6	102	107	112	116	125
90	59.2	61.8	65.6	69.1	73.3	80.6	89.3	98.6	108	113	118	124	128	137
100	67.3	70.1	74.2	77.9	82.4	90.1	99.3	109	118	124	130	136	140	149

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 1 (1966), Table 8, pages 137 and 138, by permission.

APPENDIX G

n_2 / n_1	9	10	11	12	13	14	15	16	17	18	19	20
2	0	0	0	1	1	1	1	1	2	2	2	2
3	2	3	3	4	4	5	5	6	6	7	7	8
4	4	5	6	7	8	9	10	11	11	12	13	13
5	7	8	9	11	12	13	14	15	17	18	19	20
6	10	11	13	14	16	17	19	21	22	24	25	27
7	12	14	16	18	20	22	24	26	28	30	32	34
8	15	217	19	22	24	26	29	31	34	36	38	41
9	17	20	23	26	28	31	34	37	39	42	45	48
10	20	23	26	29	33	36	39	42	45	48	52	55
11	23	26	30	33	37	40	44	47	51	55	58	62
12	26	29	33	37	41	45	49	53	57	61	65	69
13	28	33	37	41	45	50	54	59	63	67	72	76
14	31	36	40	45	50	55	59	64	67	74	78	83
15	34	39	44	49	54	59	64	70	75	80	85	90
16	37	42	47	53	59	64	70	75	81	86	92	98
17	39	45	51	57	63	67	75	81	87	93	99	105
18	42	48	55	61	67	74	80	86	93	99	106	112
19	45	52	58	65	72	78	85	92	99	106	113	119
20	48	55	62	69	76	83	90	98	105	112	119	127

Table of Critical Values of U in Mann-Whitney Rank-Sum Test (α = .05)

APPENDIX H

Two- Tailed Alternative			One-Tailed Alternative		
n	α = .05	α = .01	n	α = .05	α = .01
4			4		
5			5	1	
6	1		6	2	
7	2		7	4	0
8	4	0	8	6	2
9	6	2	9	8	3
10	8	3	10	11	5
11	11	5	11	14	7
12	14	7	12	17	10
13	17	10	13	21	13
14	21	13	14	26	16
15	25	16	15	30	20
16	30	19	16	36	24
17	35	23	17	41	28
18	40	28	18	47	33
19	46	32	19	54	38
20	52	37	20	60	43
21	59	43	21	68	49
22	66	49	22	75	56
23	73	55	23	83	62
24	81	61	24	92	69
25	90	68	25	101	77

Table of Critical Values for Signed Rank Test